2.2 Distributions, Density Functions and Moments

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# 2.2.1 Distributions and Probability Density Functions

* cdf
  + continuely
  + not continuous
  + empirical cumulative distribution function ECDF:
* two varible

conditionaly

# 2.2.2 Moments of a Random Variable

* n-th moment
* n-th central moment

and

* Estimation of moments. In practice one has observations of a random variable and from these one does an estimation of ’s moments of distribution. Given sample data of random variable , the sample mean of is
* and the sample variance is

The sample standard deviation is . The sample skewness is

and the sample kurtosis

# 2.2.3 The Normal Distribution

Theorem 2.1 (Central Limit Theorem) If is a random sample from a distribution of expected value given by and finite variance given by , then the limiting distribution of

# 2.2.4 Distributions of Financial Returns

* Are returns normally distributed? Almost any histogram of an asset’s return will present some bell-shaped curve, although not quite as smooth as the normal distribution. We show this empirical fact with a particular stock

Example (The log-normal distribution) A random variable has the lognormal distribution, with parameters and , if . In this case we write . The log-normal density function is given by

and the moments of the variable are

Therefore, if we assume that the simple return series is log-normally distributed with mean and variance , so that the log return series is such that

with mean and variance , we have that the respective moments for both series are related by the following equations

As a first application of this hypothesis of normal distribution for the continuously compounded returns, we show how to compute bounds to the future price of the underlying asset with a precision.

where and are the mean and variance of . Let . Then , and we have seen as a consequence of the Central Limit Theorem that for the quantile we have (go back to Example 2.3). From this we have

or, equivalently

These equations give bounds for the price at time , with a probability if taking . On real data, one makes an estimate of and from a sample of the log returns , and assumes these estimations of moments hold for the period ; that is, we must assume that the mean and variance of the log returns remain constant in time. Be aware then of all the considered hypotheses for these calculations to get the bounds in Eq. (2.34), so that estimations from real data should be taken as